Chapter 6 Specific Issues in Mathematics Teaching

6.1 Ways of Learning and Using Knowledge

In Target Oriented Curriculum, five fundamental ways of learning and using knowledge have been identified. They are: communicating, conceptualizing, reasoning, problem-solving and inquiring.

Young children learn to understand and use the spoken word before learning to write. In many daily life situations, the spoken language is a chief means of communication. It is therefore important that when pupils begin to learn mathematics, their ability to talk about mathematical ideas and to write about them should both be developed. There is a lot of written work connected with tasks and exercises. Explaining a result, describing the methods used in solving a problem or carrying out an inquiry and interpreting a graph are among the written tasks which provide opportunities for pupils to practise their writing skills in mathematics. As communication can be enhanced by visual presentation, teachers should make use of models, diagrams, charts and graphs in mathematics teaching as far as possible. Apart from using textbooks and workbooks, teachers should also provide opportunities for pupils to extract, appraise and use mathematical information from a range of other sources such as topic books, reference books, advertisements, newspaper, radio and television.

To master a skill, pupils usually have to repeat exercises of the same kind. To develop a concept they need a variety of experiences in which the concept is present in some form. These experiences have to be carefully planned and provided for the pupils in order to promote their understanding of the concepts. Furthermore, mathematical concepts are often inter-related. For example, subtraction is linked with addition, multiplication is linked with addition and division, percentages are linked with fractions and decimals. In fact, each concept is linked with many other aspects of mathematics. It is therefore very important for pupils to learn to recognise relationships between one concept and another. In teaching, teachers should place due emphasis on helping pupils to master the basic concepts of mathematics and create the links between the concepts.

Reasoning is a fundamental ability which pupils need at all stages of mathematics learning. Very often it is of the form 'If p then q'. In the classroom, pupils very often also learn through observations to induce concepts and patterns. It is important that reasoning should not be taught formally at this level, but it is necessary that reasoning should involve informal thinking, making hypotheses and validating them. Pupils should be encouraged to justify their answers and solution processes. In the following examples, pupils use their reasoning abilities to draw the conclusions:

(i) If $\frac{1}{4} + x = 1$ and $\frac{1}{4} + \frac{3}{4} = 1$, then $x = \frac{3}{4}$.

(ii) If Joan is 127 cm tall, John is 132 cm tall and Mary's height is between that of Joan and John, then Mary is between 127 cm and 132 cm tall.

(iii) All even numbers are divisible by 2. 14 is an even number, therefore 14 is divisible by 2.

(iv) As $(2 \times 3) \times 4 = 24$, $2 \times (3 \times 4) = 24$, therefore $(2 \times 3) \times 4 = 2 \times (3 \times 4)$;
as $(7 \times 5) \times 2 = 70$, $7 \times (5 \times 2) = 70$, therefore $(7 \times 5) \times 2 = 7 \times (5 \times 2)$;
...
...
From these examples, teachers could lead pupils to induce the associative law of multiplication.

The essence of mathematical education is to help pupils to learn and to use mathematical knowledge and techniques to tackle and solve problems. Both the processes associated with problem solving and the product should be emphasised. In the course of problem solving, four processes could be identified. Firstly, pupils have to understand the problem in terms of the vocabulary and concepts involved and the requirements of the problem. Secondly, they have to choose an appropriate strategy and may have to process data in carrying out the strategy. Thirdly, they have to communicate the solution or conclusions of the problem. Finally, they have to evaluate the solution or conclusions.

The various ways of learning and using knowledge need not be isolated from each other but may all be part of the same activity. There are no clear distinctions between problem solving and inquiring. Nevertheless, in broad terms it is useful to think of problem solving as an activity where pupils have to reach a solution to a given problem. In an inquiry, pupils are encouraged to use alternative strategies to arrive at reasonable conclusions. In particular, inquiring is involved in discovering and constructing knowledge through questioning or testing a hypothesis.

These five fundamental ways of learning and using knowledge: communicating, conceptualizing, reasoning, problem-solving and inquiring should be given due attention in planning and delivering the mathematics curriculum at all stages. They permeate the entire mathematics curriculum. Teachers are expected to engage pupils in mathematical tasks and exercises that help develop these process skills.

6.2 Integrative Use of the Subject

Although there are five dimensions in the whole mathematics curriculum, this partition does not mean that they are discrete. It represents only a convenient division which facilitates effective learning. In fact, concepts of mathematics are connected within a framework which is 'multi-dimensional'. Concepts in one dimension very often are linked to concepts in other dimensions. For example, nearly all concepts in the dimensions Measures, Algebra, Shape & Space and Data Handling are inevitably connected with concepts in the Number dimension. Furthermore, solving problems in daily life contexts, which features prominently in the mathematics curriculum at Key Stages 1 and 2, requires a repertoire of concepts

drawn from different dimensions. Teachers are therefore encouraged to make use of/set tasks, where appropriate, which call for the integrative use of the subject in both teaching and assessment.

6.3 Some Suggested Methods

To help pupils to work towards the learning targets, mathematics teaching at all year levels should provide opportunities for learning the mathematics content as well as the ways of learning and using knowledge. These include:

- exposition by teachers
- discussion between teacher and pupils and between pupils themselves
- appropriate practical work
- consolidation and practice of fundamental skills and routines
- problem solving, including the application of mathematics to everyday situations
- investigational work

Although pupils at this Key Stage are more mature than when they were at Key Stage 1 and can handle more abstract thinking, yet to enable them to learn more easily, teachers are still encouraged to use demonstrations and concrete examples in teaching mathematical concepts and procedures. This implies the use of teaching aids that provide physical representations of ideas as far as possible. Pupils should also be provided with opportunities for experimenting with materials in order to accommodate new understandings and to discover information for themselves. In this connection, discovery learning, which emphasises active participation, is most appropriate. Besides, the language abilities of pupils are also better than when they were at Key Stage 1. Therefore apart from using simple, straightforward and familiar terms to explain and to give cues to their pupils, teachers can also introduce more mathematical terms that are suitable for their level.

6.4 Catering for Individual Differences

The effectiveness of teaching depends to a large extent on whether it caters appropriately for individual differences. Owing to the fact that pupils differ in cognitive development, learning ability, interest and background, they perform differently in learning. Teachers should recognise pupils' individual abilities and interests in planning their lessons. They should refer to the actual situation and decide on the content, the quantity and the difficulty of teaching materials. And they should arrange for an appropriate learning environment to allow teaching activities to be conducted to meet pupils' abilities and needs, and to ensure what is

being taught is lively and rich in content so that pupils' interest in learning will be enhanced. Teachers may also provide pupils with opportunities to give full play to their strengths, and offer them appropriate guidance and assistance in regard to their problems and weaknesses. Owing to the above, teachers are advised to give due consideration to various aspects such as organizing the pupils, teaching/learning, graded resources and assessment.

6.4.1 Organizing the pupils

The following two broad kinds of grouping are popularly adopted in primary schools:

- streaming, where pupils close in ability remain in the same group for all or most subjects;
- mixed ability, where each group contains a wide range of ability and stays together for all or most subjects.

Teachers find teaching in mixed ability classes harder than teaching in classes where pupils are relatively close in ability. Streaming can benefit the achievement of some pupils, notably the most able ones. However, there can be a negative impact on the self-image of those pupils placed in lower streams. No matter how the pupils are organized, it is inevitable that pupils in a class will differ in ability, need and interest. Teachers need to use selectively whole class teaching, group work and individual teaching as appropriate to the task in hand. Sometimes pupils of higher attainment can be arranged to help pupils of lower attainment.

The purpose of remedial teaching is to provide additional help to pupils whose attainment falls behind that of their classmates. There are normally three modes of remedial teaching, namely, Split Class Mode, Withdrawal Mode and Before/After School Mode. In the Split Class Mode, a Mathematics class is divided into 2 classes, with fewer pupils allocated to the class of weaker learning ability. In the Withdrawal Mode, pupils (about 6 to 8 in number) of weaker ability are grouped and taught together during certain Mathematics sessions in the week. Though remedial teaching takes place during normal Mathematics sessions in both modes, teachers are able to cater for pupils' needs in a more specific way because less pupils are involved. In the Before/After School Mode, teaching takes place before or after school, so that pupils can have more time for learning Mathematics. Whichever mode of remedial teaching is adopted, teachers will have more opportunities to identify pupils' individual weaknesses and needs. They can then devise the proper measures and provide the appropriate learning activities and learning tasks/exercises to enable pupils to build up a good foundation of mathematical concepts and establish their self-confidence and sense of achievement. By so doing, their interest and ability in learning

Mathematics will be promoted and the gap in the ability of learning Mathematics between them and their classmates can then be reduced.

Remedial teaching is an effective way to cater for pupils of lower ability or with learning difficulties. Schools should make every effort to make full use of the arrangement.

6.4.2 Teaching/learning

In TOC, learning tasks and exercises play an important role in the teaching/learning process. Teachers can cater for differences in ability or attainment by providing all pupils with the same task, but varying the amount and style of support they give, i.e. giving weaker pupils more clue, varying the type of question, working through more examples with slower pupils and so on. Alternatively, teachers may provide pupils with different tasks, graded according to difficulty, so that pupils work on tasks that match their stage of progress in learning. For weaker pupils, tasks should be relatively simple and fundamental in nature; whereas for abler pupils, tasks assigned should be challenging enough to cultivate as well as to sustain their interest in learning.

In the course of teaching, teachers should try to create a favourable learning atmosphere. By making use of daily life examples or interesting mathematical problems, teachers can motivate pupils to learn and encourage them to take an active part in learning activities. For the less able pupils, teachers should make greater use of real objects, diagrams etc. to help them understand and analyse problems with an abstract context; teachers can also simplify the more complicated problems or break them up into several parts to enable the less able pupils to solve the problems by steps. Also, teachers should as far as possible eliminate pupils' other learning obstacles, such as emotional problems, so that they may concentrate more on their learning. For the abler pupils, teachers should as far as possible arrange appropriate activities for them to explore and discover some mathematical rules by themselves, so as to help further develop and affirm their abilities. Teachers can also use questioning techniques to inspire pupils. At Key Stage 2, teachers can even add some enrichment topics (please see Appendix G) to broaden these pupils' horizon of knowledge and enhance their interest in Mathematics.

6.4.3 Graded resources

Graded resources are essential in catering for pupil differences. Teachers can cooperate and sort out resources compiled by themselves or collected from other sources for the purpose of building up a resource bank in the school for reference and use. Over time and with the cooperation of all, teachers will be able to

acquire adequate and suitable graded resources to meet the needs of individual pupils.

6.4.4 Assessment

Assessment also needs to cater for learner differences. At Key Stage 2, pupils may be assessed on the same tasks and exercises and, when necessary they may be assessed on tasks and exercises set at different levels to reflect the wider differences in their learning progress.

6.5 Resources

6.5.1 Deployment of Resources

Teachers are the prime resource. Other people such as pupils themselves, their relatives and friends, other staff members in the school and visitors can also be a resource. For example, when pupils are learning Data Handling these people can be a useful source of data.

The school premises and local environment can be a resource. Buying snacks in tuckshops, considering routes to school, the shapes of buildings, the dimension of the playground, number of chairs in the classroom and so on all provide useful contexts for learning change, direction and position, symmetrical shapes, measuring and counting respectively.

In teaching, a variety of material resources will be required:

- textbooks or learning packages
- reference books
- measuring equipment
- various apparatus designed to explore and illustrate particular concepts such as number and shape
- instruments and other equipment for drawing shapes and making models
- games and toys

Many materials such as newspapers, magazines, advertising leaflets, brochures, packaging, maps, plans and timetables and discarded household items, which pupils find in daily life, can be used to good effects as useful teaching resources.

6.5.2 The TOC Resource Centre

In order to help schools implement the Target Oriented Curriculum, the TOC Section of the Curriculum Development Institute has gathered a large quantity of related materials for teachers' reference. These are available for their use during office hours at the Target Oriented Curriculum Section, Curriculum Development Institute, Education Department, 7/F To Kwa Wan Market and Government Offices, 165 Ma Tau Wai Road, To Kwa Wan, Kowloon. The Mathematics Teaching Centre, Advisory Inspectorate, also has reference materials available for teachers' use during 9:00 am - 12:30 pm on Mondays, 2:00 pm - 5:00 pm on Wednesdays and 9:00 am - 12:00 noon on Saturdays (except on public holidays) at Room 310, 4 Pak Fuk Road, North Point, Hong Kong.